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FORM 3SBP

1996

The College Board

Advanced Placement Examination

CALCULUS BC

SECTION II

This green insert may be used for reference and/or scratchwork as you answer the free-response questions, but be sure to show all your work and your answers in the pink booklet. No credit will be given for work shown on this green insert.

CALCULUS BC

SECTION II

Time — 1 hour and 30 minutes

Number of problems — 6

Percent of total grade — 50

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUC

General instructions for this section are printed on the back cover of the test booklet.

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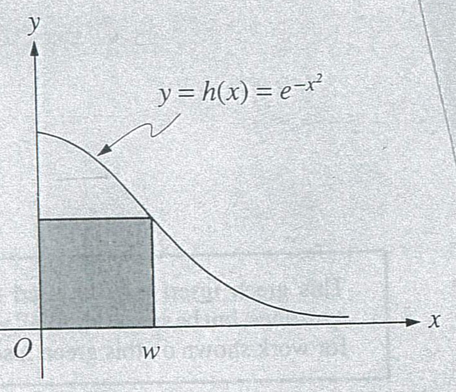
1. Consider the graph of the function h given by $h(x) = e^{-x^2}$ for $0 \le x < \infty$.

   (a) Let R be the unbounded region in the first quadrant below the graph of h. Find the volume the solid generated when R is revolved about the y-axis.

   (b) Let $A(w)$ be the area of the shaded rectangle shown in the figure to the right. Show that $A(w)$ has its maximum value when w is the x-coordinate of the point of inflection of the graph of h.



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2. The Maclaurin series for $f(x)$ is given by $1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \ldots + \frac{x^n}{(n+1)!} + \ldots$

   (a) Find $f'(0)$ and $f^{(17)}(0)$.

   (b) For what values of x does the given series converge? Show your reasoning.

   (c) Let $g(x) = xf(x)$. Write the Maclaurin series for $g(x)$, showing the first three nonzero terms and the general term.

   (d) Write $g(x)$ in terms of a familiar function without using series. Then, write $f(x)$ in terms of the same familiar function.

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$$\lim_{n\to\infty} \left| \frac{\frac{x^{n+1}}{(n+1+1)!}}{\frac{x^n}{(n+1)!}} \right| = \left| \frac{x}{(n+1+1)!} \cdot (n+1)! \right|$$

3. The in t

(a)

3. The rate of consumption of cola in the United States is given by $S(t) = Ce^{kt}$, where S is measured in billions of gallons per year and t is measured in years from the beginning of 1980.

(a) The consumption rate doubles every 5 years and the consumption rate at the beginning of 1980 was 6 billion gallons per year. Find C and k.

(b) Find the average rate of consumption of cola over the 10-year time period beginning January 1, 1983. Indicate units of measure.

(c) Use the trapezoidal rule with four equal subdivisions to estimate $\int_5^7 S(t)\,dt$.

(d) Using correct units, explain the meaning of $\int_5^7 S(t)\,dt$ in terms of cola consumption.

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4. This problem deals with functions defined by $f(x) = x + b \sin x$, where b is a positive constant and $-2\pi \le x \le 2\pi$.

(a) Sketch the graphs of two of these functions, $y = x + \sin x$ and $y = x + 3 \sin x$, as indicated below.

Note: The axes for these two graphs are provided in the pink test booklet only.

(b) Find the x-coordinates of all points, $-2\pi \le x \le 2\pi$, where the line $y = x + b$ is tangent to the graph of $f(x) = x + b \sin x$.

(c) Are the points of tangency described in part (b) relative maximum points of f? Why?

(d) For all values of $b > 0$, show that all inflection points of the graph of f lie on the line $y = x$.

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x+3sinx

1+3cosx

cosx = −1/3

x = −0.390

3.48

−2.80

5.94

±1.9

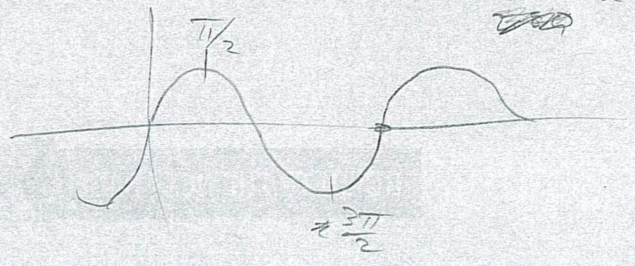
±4.37

d/dx x+sinx π

1 + cosx = 0

cos x = −1

x = −π, π etc.



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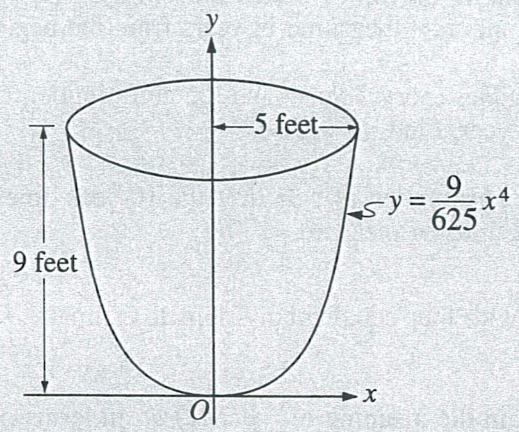
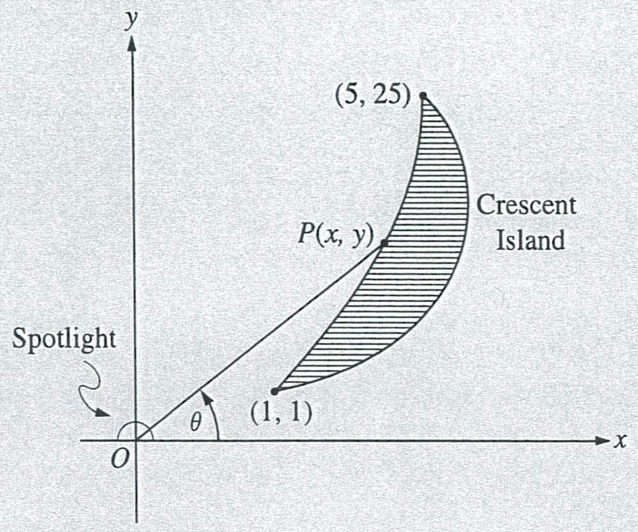


5. An oil storage tank has the shape shown above, obtained by revolving the curve $y = \frac{9}{625}x^4$ from $x = 0$ to $x = 5$ about the y-axis, where x and y are measured in feet. Oil weighing 50 pounds per cubic foot flowed into an initially empty tank at a constant rate of 8 cubic feet per minute. When the depth of the oil reached 6 feet, the flow stopped.

   (a) Let h be the depth, in feet, of oil in the tank. How fast was the depth of the oil in the tank increasing when $h = 4$? Indicate units of measure.

   (b) Find, to the nearest foot-pound, the amount of work required to empty the tank by pumping all of the oil back to the top of the tank.



Note: Figure not drawn to scale.

6. The figure above shows a spotlight shining on point $P(x, y)$ on the shoreline of Crescent Island. The spotlight is located at the origin and is rotating. The portion of the shoreline on which the spotlight shines is in the shape of the parabola $y = x^2$ from the point (1, 1) to the point (5, 25). Let θ be the angle between the beam of light and the positive x-axis.

   (a) For what values of θ between 0 and 2π does the spotlight shine on the shoreline?

   (b) Find the x- and y-coordinates of point P in terms of $\tan\theta$.

   (c) If the spotlight is rotating at the rate of one revolution per minute, how fast is the point P traveling along the shoreline at the instant it is at the point (3, 9) ?

END OF EXAMINATION